FAST ITERATIVE ADAPTIVE RECONSTRUCTION IN LOW-DOSE CT IMAGING

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ABSTRACT

Traditional iterative tomographic reconstruction methods resort to gradient decent methods and require significant computation due to slow convergence. We divide the iterative reconstruction into two stages in the image and Radon spaces, respectively. First, we refine the reconstruction result by image space adaptive filtering. This finds a feasible update direction based on signal modeling. Second, we minimize the discrepancy between the sinograms along the update direction in Radon space and guarantee convergence. Reconstruction from clinical data using the proposed algorithm converges extremely fast and provides satisfactory reconstruction results in far fewer iterations than traditional methods.

Index Terms— Image reconstruction, medical imaging, adaptive filters, medical diagnosis, iterative methods, tomography, optimization methods, Radon transforms

1. INTRODUCTION

Tomographic image reconstruction techniques are of great interest to medical imaging, sonar/radar imaging, product inspection and geophysical imaging. Tomographic imaging reconstructs an image from a set of projection measurements that are described by the *Radon* Transform [1]. However, the number and quality of the projections is often limited in practice. For example, much of the recent attention is directed to low-dose diagnostic tomographic imaging to reduce the risks of excessive radiation, which greatly restricts the number and the quality of the projection measurements. Non-iterative algorithms, such as Filtered Back Projection (FBP) [1, 2], cannot handle this situation well and produce significant streak artifacts. Interpolation in Radon space is proposed to reduce artifacts [3, 4]. However, the prior knowledge dealing with the null space of the projections is usually more difficult than signal modeling in the image space.

Iterative methods [5] usually achieve better results by incorporating prior knowledge in the image space to regularize the reconstruction in an iterative optimization scheme. The objective function typically consists of a data fidelity term (matching between the measured sinogram and the forward projections of the reconstructed image) and a regularization term (e.g., smooth surface with edge modeling). However, the

optimization is usually based on the gradient decent method. The derivative on the data fidelity term results in a back projection in each iteration, which provides blurry results and converges slowly [5]. Furthermore, the step size to update in the gradient-decent direction is usually set small due to convergence issues, which further slows down the convergence.

In this paper, we focus on low-dose Computerized Tomography (CT) and propose a novel two-stage framework. Instead of optimizing the data fidelity term and the regularization term simultaneously, we divide the reconstruction into two simpler optimization steps, similar to the alternating minimization algorithm in blind image deblurring [6]. First, we find a good direction to update the reconstruction image to better fit the observed sinogram. FBP based on the residual sinogram provides a better update direction than back projection. But simple FBP might not satisfy the prior signal models (e.g., significant streak artifacts with restricted number of projections). We propose an adaptive filter in the image space to enforce the regularization term and refine the FBP result into signal subspace. Second, given the update direction in signal subspace (i.e., the refined FBP result), we minimize the residual error between the sinograms by projecting the residual sinogram to the Radon transform of the update direction. Optimal step size can be found to update the reconstruction and minimize the mean square error of the residual sinogram. The iterative process continues until convergence. This new framework can always find the optimal step size to update the reconstruction and guarantees convergence. Experiments from clinical low-dose CT data demonstrate that the new framework converges in just a few iterations. In addition, it provides a much better result for a restricted number of iterations than traditional iterative methods.

This paper is organized as follows. Section 2 describes the proposed two-stage algorithm. Section 3 discusses the detailed implementation of the adaptive image filtering in the image space. Section 4 evaluates the algorithm and conclusions are given in Section 5.

2. TWO-STAGE ITERATIVE RECONSTRUCTION

Traditional iterative methods discretize the Radon transform operator to form weight factors between each image pixel

intensity and each sinogram value. This contribution set is the system matrix A (i.e., forward projection). By stacking the known sinogram values into a vector b , the reconstruction problem can be formulated as solving a linear system of a vector x representing the unknown image to be reconstructed: $Ax = b$. Regularization based on signal properties (e.g., smooth surface and edge modeling) is usually necessary to assure valid solutions due to the ill-posed nature of inverse problems. The objective function can be designed as follows

$$J = \|Ax - b\|^2 + \alpha \|Rx\|^2 \quad (1)$$

where $\|Ax - b\|^2$ is the data fidelity term and α is the weight of the regularization term $\|Rx\|^2$. The optimization is usually done by gradient decent methods based on the derivative

$$\partial J / \partial x = (A^T A + \alpha R^T R)x - A^T b \quad (2)$$

where A^T represents the back projection operator. Unfortunately, back projection derived from the derivative cannot pick up the high frequency structures in the image effectively and suffers from slow convergence [5].

Our objectives are in two-folds. On one hand, we need to find out how to update the reconstructed image based on the regularization term. Various regularization methods (e.g., Markov Random field or Bayesian restoration techniques) have been extensively studied and usually can enforced be more effectively in the image space rather than Radon space. On the other hand, we also need to match the sinograms, which can be done in Radon space. Given a feasible update direction, this optimization step can be done effectively by projecting the residual sinogram to the update direction in Radon space and find the optimal step size. Traditional methods address these two parts simultaneously in Radon space and hence are difficult to optimize.

Therefore, we propose to divide this optimization problem into two stages and handle them in the image and Radon spaces, respectively. In the first stage, to refine the reconstructed image in each iteration, we try to find a feasible update direction (based on regularization term R in Equation (1)) in the image space. We employ FBP to reconstruct an image $I_{(i)}$ from the sinogram residue $sino_{(i)}$ in the i^{th} iteration (the initial sinogram residue is the original sinogram, $sino_{(0)}$). But, the induced streak artifacts under low-dose imaging do not satisfy the prior constraints and are hard to correct in successive iterations. The regularization term R provides good signal modeling for adaptively filtering the artifacts while preserving sharp object boundaries. The artifact reduced image $I_{(i)AR}$ represents a refined feasible update direction in the signal subspace. A detailed implementation of this stage based on adaptive bilateral filtering is described in Section 3.

In the second stage, the Radon transform of $I_{(i)AR}$ provides the update direction in Radon space $sino_{(i)AR}$ to enforce the data fidelity constraint, producing a new sinogram

residue estimate $sino_{(i+1)}$ with a step size $a_{(i)}$. Based on the linearity of the Radon transform, we have

$$A(I + a_{(i)}I_{(i)AR}) = A(I) + a_{(i)}A(I_{(i)AR}) \quad (3)$$

We update the reconstruction results I as follows

$$I \leftarrow I + a_{(i)}I_{(i)AR} \quad (4)$$

and the new sinogram residue estimate $sino_{(i+1)}$ will be

$$sino_{(i+1)} = sino_{(i)} - a_{(i)}sino_{(i)AR} \quad (5)$$

Since the update direction (i.e., $I_{(i)AR}$) has already been refined based on the regularization term, the remaining task is to find the optimal $a_{(i)}$ to minimize the data fidelity term

$$\bar{J} = \|sino_{(i)} - a_{(i)}sino_{(i)AR}\|^2 \quad (6)$$

$a_{(i)}$ can be obtained by setting the derivative of (6) to zero

$$a_{(i)} = (sino_{(i)}^T sino_{(i)AR}) / \|sino_{(i)AR}\|^2 \quad (7)$$

This two-stage framework can be illustrated by Figure 1. It ensures a sinogram correction procedure that always reduces discrepancy in Radon space and hence guarantees convergence. The final reconstruction result is

$$I = a_{(0)}I_{(0)AR} + \dots + a_{(i)}I_{(i)AR} + \dots \quad (8)$$

Fig. 1. Updating scheme.

3. IMAGE SPACE REGULARIZATION

In the previous section, we explained the overall steps for optimizing the two-stage framework. This section presents a detailed implementation for image space regularization. The Radon space update ensures a sinogram correction procedure that satisfies the data fidelity. FBP relates it to image space and provides an image that matches the sinogram residual. In addition, FBP picks up high frequency components much faster compared to back projection, hence functional handling with edges and other regions imposed by the regularization term becomes easier in image space. However, the FBP result does not satisfy the prior signal modeling (e.g., smoothness constraint) and cannot be used directly without additional regularization. Therefore, the proposed paradigm employs image space adaptive filtering to enforce the regularization term

(i.e., signal modeling to guarantee valid reconstruction results). Based on smoothness constraints in a Bayesian image restoration context, this step aims to restore an image $\hat{I}$ (instead of use FBP result directly) that optimizes the following objective function, the subscript indicates signal estimated at pixel p

$$\hat{I} = \arg \min_{\hat{I}} \sum_p (\lambda_1 (\hat{I}_p - I_{fbp,p})^2 + \lambda_2 (1 - e_p) (\hat{I}_p - \bar{I}_{fbp,p})^2 + \lambda_3 e_p) \quad (9)$$

where $\lambda_i (i = 1, 2, 3)$ weights each constraint and $\bar{I}_{fbp,p}$ is the average intensity of the true signal in a small neighborhood. The first term enforces that the estimated signal should look like the observed image. The second term models the signal property and prefers a smooth signal. e_p represents a pixel-based edge modeling process which varies between 0 and 1. When $e_p = 1$, it disables the smoothness constraint and hence preserves sharp edges. A penalty λ_3 is introduced to prevent treating all pixels as edges. Edge modeling employs a sobel edge detector to approximate spatial gradient at each pixel location to identify edges. From the initial FBP result, we obtain the object boundaries with artifacts. Then, we tune the sobel edge detector, adjusting the different sizes and thresholds accordingly to identify the edges of the object structures. Not surprisingly, if the above process is revisited during another iteration, the threshold varies. Thus, we use an adaptive threshold to solve this. Based on the observation that artifacts are hard to remove compared to weak object structures (which can be recovered in future iterations), we manually tune the threshold in the first iteration. This removes artifacts as much as possible at the expense of removing some weak structures as well. At the same time, we estimate the histogram of the sobel detection to find the Cumulative Density Function (CDF) and compute the percentile based threshold. In future iterations, we use this percentile as an adaptive threshold, instead of manually tuning the threshold in every iteration.

In the second term of (9), using predetermined weights for averaging to achieve a smooth signal, $\bar{I}_{fbp,p}$, causes blurring in the vicinity of edges. Many schemes (e.g., nonlinear diffusion, adaptive bilateral algorithms [7]) are designed to optimize this. A detailed discussion showing the connection of the bilateral algorithm to nonlinear diffusion and other adaptive filtering schemes can be found in [8]. Here we choose the bilateral algorithm for obtaining $\bar{I}_{fbp,p}$ because it is efficient and non-iterative. While normal low pass filtering averages the neighboring pixel intensity values with decreasing weights for pixels at larger distances, a bilateral filter adds a second dimension to measure radiometric similarity. This dimension assigns greater weights to those neighboring pixels with an intensity that is similar to the center pixel intensity. In a space variable representation, we obtain a bilateral filtered image $I_{bil}(\vec{r})$ from input FBP reconstruction $I_{fbp}(\vec{r})$ using

gaussian realization of both filtering components

$$I_{bil}(\vec{r}) = c \int \exp\left(-\frac{|\vec{\xi} - \vec{r}|^2}{2\sigma_1^2}\right) \times \exp\left(-\frac{|I_{fbp}(\vec{\xi}) - I_{fbp}(\vec{r})|^2}{2\sigma_2^2}\right) I_{fbp}(\vec{\xi}) d\vec{\xi} \quad (10)$$

where c is the normalization factor. σ_1 controls normal spatial low pass filtering, while σ_2 controls the discrimination. Smooth regions share similar pixel intensities across their neighborhood, so the pixel intensity differences from artifacts are much smaller than a tuned σ_2 , making the second exponential close to 1, thus bilateral filtering smoothes away artifacts. However, when the filtering operation reaches pixels at the edges, the intensity differences of neighboring pixels in smooth regions are significantly larger than σ_2 , making the second exponential close to 0, which cancels the contributions from those pixels and adaptively achieves an intensity-selective filtering. In this way, artifacts are removed without blurring the edges.

We describe our approach in combining the terms in (9). To achieve the smoothness constraint efficiently in image space, we model the object being reconstructed in terms of its boundary and define a pixel based weighting function e_p based on edge detection threshold th , which is adapted from what pixel intensity represents in image space. To preserve edges, e_p weights the original FBP reconstruction result I_{fbp} more on the structure boundaries. At the same time, to remove the artifacts, e_p weights the bilateral-filtered reconstruction I_{bil} more on non-boundary parts. In this way, we produce the final artifact-reduced image of this step

$$I_{AR} = \hat{I} = (1 - e_p)I_{bil} + e_p I_{fbp} \quad (11)$$

e_p is chosen to be a tunable function of intensity difference from threshold, and is bounded between 0 and 1

$$e_p = \frac{1}{1 + e^{k(th - I_{fbp,p})}} \quad (12)$$

where k tunes the steepness of the function, $I_{fbp,p}$ refers to the intensity of pixel p in image I_{fbp} .

4. EVALUATION

In order to validate the effectiveness of the proposed method, we evaluate the algorithm with a clinical low-dose CT P30A sinogram with 100 projections and 367 detectors per projection (as shown in Figure 2(a)). The dimensions of the reconstructed image are 258 by 258. Filtered back projection of the original sinogram yields an unacceptable reconstructed image (Figure 2(b)) with streak artifacts (Figure 2(c)).

We present experimental results for the proposed method and compare it with the traditional iterative method (i.e., one objective function with both the data fidelity and regularization terms) optimized by gradient decent method. To make

them comparable, the step size in each iteration is optimized for both methods. The energy of the residual sinogram in Radon space serves as the quantitative metric to judge the convergence. The results shown in a logarithmic scale in Figure 2(d) proves that the proposed algorithm converges much faster and achieves a better solution within a restricted number of iterations. For visual comparison, the traditional iterative scheme after 10 iterations (Figure 3(a)) shows a blurred reconstruction. Reconstruction after 50 iterations using the traditional iterative scheme provides a more reasonable reconstruction (Figure 3(b)), but it still cannot beat the proposed method using just 10 iterations (Figure 3(c)). In addition, the bilateral filtering in the proposed framework is much faster than the back projection procedure and does not increase the computation of each iteration dramatically. With significantly faster convergence rate, the proposed algorithm yields an improved reconstruction and dramatic computational savings without sacrificing spatial resolution.

Fig. 2. (a) Original sinogram; (b) FBP reconstruction; (c) Streaking artifacts; (d) Performance comparison.

5. CONCLUSION

We have developed a fast two-stage iterative framework for tomographic image reconstruction which combines the benefits of the edge preserving artifact reduction and efficient optimization. This has been achieved by adaptive bilateral filtering to modulate the FBP reconstruction in image space, which generates a refined update direction regularized by prior constraints. Data fidelity in Radon space is achieved by projecting the measured sinogram to the Radon transform of the modulated FBP. The proposed framework finds the optimal step size to reduce the sinogram residual. Evaluation of clinical images demonstrates the advantages and effectiveness of our approach.

Fig. 3. (a) Traditional iterative method after 10 iterations; (b) Traditional iterative method after 50 iterations; (c) The proposed algorithm after 10 iterations.

6. REFERENCES

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